

# The Electromagnetic Form Factor for the Kaon in the Light-Front Approach

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**Abstract.** In this paper, the kaon electromagnetic form factor is calculated with a light-front constituent quark model (LFCQM). The electromagnetic components of the current (like the component " $J_K^+$ ") are extracted from the Feynman triangle diagram with the light-front approach. By means of the Feynman diagram, we also obtained the electroweak decay constant and the charge radius for the kaon in the light-front approach. In this work, the kaon observables are calculated and a fairly good agreement is obtained with a very higher accuracy when compared with the experimental data.

**Keywords:** Light-Front, QCD, quark model, electromagnetic current, electromagnetic form factor, kaon

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With the light-front constituent quark model, *LFCQM*, is possible to give answers to hadronic physics, in terms of the freedom degrees from *QCD*, i.e., quarks and gluons [1]. In this work, the main proposal with the light-front is to describe in a consistent way the hadronic bound state composite system  $q\bar{q}$  (the kaon meson) and the corresponding electromagnetic (e.m.) form factor. In the last years, the pion e.m. form factor has been calculated in many works (see the references [2, 3, 4] and the references therein); but for the kaon case, there were few theoretical works [5, 6, 7]. The extraction of the electromagnetic form factor in the light-front approach depends on which component of the electromagnetic current (e.c.) is utilized to calculate the form factors, due to problems related to the rotational symmetry breaking [8, 9, 10, 11, 12]. The e.c. with the light-front approach has another contribution, besides the valence contribution to the electromagnetic current. That contribution corresponds to the pair terms added to the matrix elements of the e.c. [8, 10, 13]. In this work, we report results for the kaon e.m. form factor that are extracted from the plus component of the e.c. in light-front formalism,  $J_K^+ = J^0 + J^3$ , with a pseudoscalar coupling of the quarks, and considering the Breit frame ( $q^+ = 0$ ,  $q_\perp = (q_x, 0) \neq 0$ ). In the case of  $J_K^+$ , there is no pair term contribution in the Breit frame. However, for the  $J_K^-$  component of the e.c., the pair term contribution is different from zero and necessary in order to preserve the rotational symmetry of the current. Then, the matrix elements of the e.c. with the light-front approach have other contributions, besides the valence contribution to it (see the references [6, 9, 12] and the references therein).

The studies concerning light vector and scalar mesons are important because they give a way to try understanding why QCD works in the non-perturbative regime and besides why the pseudoscalar mesons are the observed light hadrons related with the

chiral symmetry breaking.

The kaon e.m. form factor is calculated from the Feynman triangle diagram in the impulse approximation. The e.c.  $J^\mu$  can be written in terms of the quark fields  $q_f$  and charge  $e$  as:

$$J_q^\mu(q^2) = -i2e_q \frac{m^2}{f_K^2} N_c \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ S(k) \gamma^5 S(k-p') \gamma^\mu S(k-p) \gamma^5 \right] \Gamma(k, p') \Gamma(k, p) ,$$

$$J_{\bar{q}}^\mu(q^2) = q \leftrightarrow \bar{q} \text{ in } J^\mu(q^2) , \quad (1)$$

in which  $N_c = 3$  is the numbers of colors and  $e_q$  is the quark (anti-quark) charge. In the equation above, the fermion propagators are  $S(p, m) = 1/(p - m^2 + i\epsilon)$ . The calculation are performed in the Breit frame, ( $q^+ = 0$ ), with  $p^\mu = (0, -q/2, 0, 0)$  and  $p'^\mu = (0, q/2, 0, 0)$ , for the initial and final momenta of the system, respectively. The momentum transfer is  $q^\mu = (0, q, 0, 0)$  and  $k^\mu$  is the spectator quark momentum. The factor 2 appears from the isospin algebra. The function  $\Gamma(k, p)$  is the regulator vertex function, used in order to regularize the Feynman triangle diagram, Eq.(1), for the current. We have utilized as  $q\bar{q}$  vertex function the nonsymmetric vertex  $\Gamma^{NSY}(k, p) = N/((p-k)^2 - m_R^2 + i\epsilon)$  [9]. The matrix elements of the e.c. for the kaon can be related with the e.m. form factor by means of the equation

$$(p + p')^\mu F_K(Q^2) = \langle K(p') | J^\mu | K(p) \rangle , \quad (2)$$

where  $K$  is the kaon field operator and  $Q^2 = -q^2$ . The final covariant e.m. form factor for the kaon can be obtained from the Eq. (1) and Eq. (2):

$$F_K(q^2) = -\frac{i2eN_c}{(2P)} \frac{m^2}{f_K^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ S(k) \gamma^5 S(P' - k) \gamma^\mu S(P - k) \gamma^5 \right] \Gamma(P', k) \Gamma(P, k) . \quad (3)$$

The  $J_K^+$  component of the e.c. is utilized in order to extract the kaon e.m. form factor. The trace in Eq.(3) is the sum of the two parts

$$\begin{aligned} \text{Tr}[1] &= \left[ \gamma^5 (\not{k} - \not{P} + m_u) \gamma^\mu (\not{k} - \not{P}' + m_u) \gamma^5 (\not{k} + m_{\bar{s}}) \right] , \\ \text{Tr}[2] &= \left[ \gamma^5 (\not{k} - \not{P} + m_{\bar{s}}) \gamma^\mu (\not{k} - \not{P}' + m_{\bar{s}}) \gamma^5 (\not{k} + m_u) \right] . \end{aligned} \quad (4)$$

The final trace is  $\text{Tr}_{\text{kaon}}[\dots] = (2/3) \text{Tr}[1] + (1/3) \text{Tr}[2]$ , where the factors 2/3 and 1/3 are isospin factors. The quadri-momentum integration of the Eq.(3) has two intervals contribution: the first (I) is on  $0 < k^+ < P^+$  and the second (II) is on  $P^+ < k^+ < P'^+$ , where  $P'^+ = P^+ + \delta^+$ . The first interval is the contribution of the valence wave function for the e.m. form factor and the second interval corresponds to the pair terms contribution to the matrix elements of the e.c. [8, 9, 10, 13, 14]. In the case of the nonsymmetric vertex with the plus component of the e.c., the second interval does not give any contribution for the matrix elements of the current, because the non-valence or pair terms contribution in this case is zero [6, 9]. One can verify that only the on-shell pole  $\bar{k}^- = \frac{f_1 - i\epsilon}{k^+}$  contributes

to the  $k^-$  integration in the interval (I),  $0 < k^+ < P^+$ . So, after the Cauchy integration in the light-front energy  $k^-$ , the equation for the kaon e.m. form factors with nonsymmetric vertex is

$$F_{\bar{q}}^+(q^2) = e_q \frac{N^2 g^2 N_c}{P^+} \int \frac{d^2 k_\perp dx}{2(2\pi)^3 x} \left[ -4 \left( f_1 x P^+ - x P^+ k_\perp^2 - 2 f_1 P^+ + 2 k_\perp^2 P^+ - \frac{x P^+ q^2}{4} \right) - \frac{4 f_1 P^+}{x} + 8 P^+ (x-1) m_q m_{\bar{q}} - 4 x P^+ m_q^2 \right] \theta(x) \theta(1-x) \Phi_f^*(x, k_\perp) \Phi_i(x, k_\perp) .$$

$$F_{\bar{q}}^+(q^2) = q \leftrightarrow \bar{q} \text{ in } F_q^+(q^2) , \quad (5)$$

where  $f_1 = k_\perp^2 + m_{\bar{q}}^2$ ,  $f_2 = (P-k)_\perp^2 + m_q^2$ ,  $f_3 = (P'-k)_\perp^2 + m_q^2$ ,  $f_4 = (P-k)_\perp^2 + M_R^2$  and  $f_5 = (P'-k)_\perp^2 + M_R^2$ . The light-front wave function for the kaon with the nonsymmetric vertex is written like:

$$\Psi_q^i(x, k_\perp) = \left[ \frac{N}{(1-x)^2 (m_{K^+}^2 - M_0^2) (m_{K^+}^2 - M_R^2)} \right] , \quad (6)$$

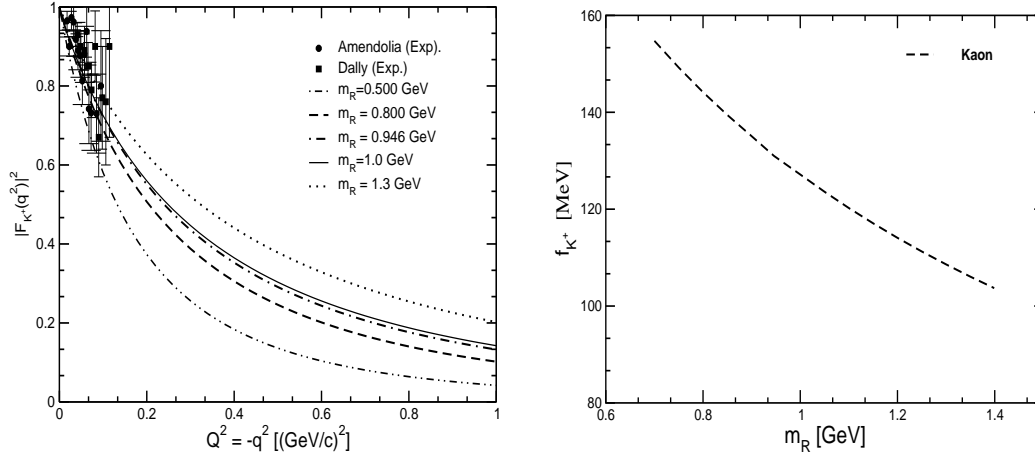
where  $x = k^+ / P^+$  is the momentum fraction carried by the quark and  $0 < x < 1$ . The  $M_R^2$  function is the square mass operator

$$M_R^2 = M^2(m_{\bar{q}}, M_R) = \frac{k_\perp^2 + m_{\bar{q}}^2}{x} + \frac{(P-k)_\perp^2 + M_R^2}{1-x} - P_\perp^2 . \quad (7)$$

The free quark square mass operator is given by  $M_0^2 = M^2(m_q^2, m_q^2)$ , ( $\bar{q} \leftrightarrow q$ ). The normalization constant  $N$  is found with the charge conservation condition  $F_{K^+}^+(0) = 1$ . In the light-front approach, besides the valence contribution to the e.c., the non-valence components give contribution to the e.c. [9, 10, 11], but in the case of the e.c. for the kaon calculated with the plus component of the current, the pair terms contribution is zero [6]. The calculation of the kaon e.m. form factor in the light-front with  $J_K^+$  gives the same result as the covariant one [6]. The parameters utilized are the constituent quark masses  $m_q = m_u = 0.220$  GeV,  $m_{\bar{q}} = m_{\bar{s}} = 0.508$  GeV and the regulator mass  $m_R = 1.0$  GeV, which are adjusted to fit the electromagnetic radius of the kaon. In the case of the nonsymmetric vertex, the kaon radius is utilized to fix the parameters of the model. The kaon mass utilized is the experimental value,  $m_K = 0.496$  GeV [15].

With these parameters, the calculated electromagnetic radius of the kaon is  $\langle r_{K^+} \rangle = 0.636$  fm and  $f_{K^+} = 126.9$  MeV, close to experimental values,  $\langle r_{K^+} \rangle^{exp} = 0.560$  fm and  $f_{K^+}^{exp} = 110.4$  MeV [16], and the e.m. form factor for the kaon is shown in the left frame of figure 1. Further, in the right frame of figure 1, we show the calculated decay constant as function of the regulator mass  $m_R$ . Due to the fact that  $J_K^+$  does not have light-front pair term contributions, the kaon e.m. form factor result is equal to the one obtained in a covariant calculation. At very low momentum transfer, the light-front model presented here gives better agreement with experimental data [15, 16].

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**FIGURE 1.** The left frame shows the kaon e.m. form factor calculated with the model presented here for different values of  $m_R$  and compared with the experimental data [16]. The results are covariant and free of the zero modes contributions (see the ref. [2] and references therein). The right frame shows the kaon decay constant for different values of  $m_R$ .

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